USE OF CRITERIA NPV AND IRR FOR CHOOSING
AMONG INVESTMENT PROJECTS

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Abstract. Two of the most important criteria are the net present value (NPV) and the internal rate of return (IRR) for choosing among investment projects. In many circumstances, investment projects are ranked in the same order by both criteria. In some situations, however, the two criteria provide different rankings. The debate is an old one (e.g. going back to Böhm-Bawerk, 1884). Let us explain the essence of the NPV and IRR indicators. The basis of economic calculations in the field of investment is the idea that a cash euro today is more valuable than a euro promised in a year. If a bank lends N euros to an entrepreneur today, then in a year the bank demands to return $N(1 + E)$ euros, where E is the bank interest. Another type of calculation is carried out by the entrepreneur. If he invests N euros in some project today, then in a year he hopes to receive $N(1 + IRR)$ euros, where IRR is the internal rate of return of the project implemented by the entrepreneur. Naturally, the value of IRR is only an assumed, indicative, and the entrepreneur is expecting IRR more than E. The present work arose from discussions of the results of the French economist Pierre Masse "Le Choix des investissements, critères et méthodes" published in 1959. The main goal of the paper is to give the proof of both IRR and NPV formulas (in a particular simplified case) and a geometric interpretation of these very complex equations (useful for the training purpose, at least). The analysis of IRR and NPV indicates an unequivocal choice among the criteria NPV and IRR. This confirms a simple numerical example on the fallacy of Masse's IRR reasoning. No unambiguous solution has been found yet. It can be explained if we allow that the bank interest relates to Macroeconomics, largely concerned with nation scale projects but the entrepreneur interest relates to Microeconomics, to internal rate of return. The world continues to search for a single consistent criterion for evaluating investments.

Keywords: investment, net present value, internal rate of return.

Introduction

Long-term investments in rapidly changing conditions, especially conditions that change or may change at any time due to new products and technologies, "is like shooting at a target that is not only indistinct but moving − and moving jerkily at that," − these words belong to Joseph Schumpeter (1883-1950) said by him in his old age − after a long life devoted to the study of the economic development of society and, above all, entrepreneurial activity, the basis of which is success in investments [1]. Speaking about the investment mechanism, J. Schumpeter calls profit maximization as the goal of entrepreneurial activity, however, he does not give clear recipes on the basis of which entrepreneurs should give preference to one or another project − whether in terms of the expected total profit or another criterion [2]. Schumpeter notes an essential feature of investments, namely, their long-term nature, which leads to a difference in costs and results. At the same time, he refers to Böhm-Bawerk [3] as the eminent specialist in the issues of accounting for the time factor: Böhm-Bawerk sought to explain interest as a reward of the kind claimed by all who supply their own productive services or their land, long before there can be a product to be consumed.

Two of the most important criteria are the net present value (NPV) and internal rate of return (IRR) for choosing among investment projects. In many circumstances, investment projects are ranked in the same order by both criteria. In some situations, however, the two criteria provide different rankings. A difference between rankings implies inconsistent recommendations about the "best project". Let us explain the essence of the NPV and IRR indicators. The basis of economic calculations in the field of investment is the idea that a cash euro today is more valuable than a euro promised in a year. If a bank lends N euros to an entrepreneur today, then in a year the bank demands to return $N(1 + E)$ euros, in two years $N(1 + E)^2$ euros, etc., where E is the bank interest. Another type of calculation is carried out by the entrepreneur. If he invests N euros in some project today, then in a year he hopes to receive $N(1 + IRR)$ euros, in T years $N(1 + IRR)^T$ euros, where IRR is the internal rate of return of the project implemented by the entrepreneur. Naturally, the value of IRR is only an assumed, indicative, and the entrepreneur is expecting IRR more than E.

If Q_t is the expected return on investment, i.e. the difference between revenue and costs in the t-th year, T is the settlement period, and E is the amount of bank interest, then the discounted total profit NPV is determined by the formula:

$$NPV = \sum_{t=0}^{T} Q_t \frac{1}{(1+E)^t} . \tag{1}$$

For the same investment project, its rate of efficiency (internal rate of return IRR) is defined as the root of the equation

$$\sum_{t=0}^{T} Q_t \frac{1}{(1+IRR)^t} = 0 . \tag{2}$$

The present work arose as some comments on the Pierre Massé book [4] published in 1959. Pierre Massé (1898-1987) was an economist, engineer, applied mathematician, and a high official in the French government. As criteria for evaluating the effectiveness of investments, P. Massé uses both NPV and IRR criteria, giving preference to the first one. He claims that the ordering of investment options depending on the value of their discounted total profit NPV was used in the works of I. Fisher (1867-1947), J. M. Keynes (1883-1946), J. R. Hicks (1904-1989), P. Samuelson. Other authors used the internal rate of return IRR criterion: E. Böhm-Bawerk (1851-1914), K. Wickzell (1851-1926), F. H. Knight (1885-1972), K. E. Bouldang (1910-1993), F. A. Hayek (1899-1938). P. Massé stated: "In this book, we have preferred the discounted total profit criterion, considering the public interest, but we do not deny the usefulness of the concept, the norms of investment efficiency with its correct definition" [4].

We analyzed the works of the world known economists named by Pierre Massé among the supporters of the NPV criterion [5] and did not find evidence that entrepreneurial activity should be guided by "the ordering of investment options by the value of their discounted total profit NPV", as Pierre Massé argues. This unsubstantiated Massé's claim suggested the topic of the article.

The rest is structured as follows. At first, the proof of both IRR and NPV formulas is given. Then we consider a geometric interpretation of these formulas. Some criticism of Massé's reasoning about the preference for the NPV criterion and a simple numerical example on the fallacy of Massé's IRR reasoning follows. And finally, contemporary data for determining an appropriate investment criterion indicates an unequivocal choice among the criteria NPV and IRR.

Proof of IRR and NPV formulas

In the section "Ordering of investments according to the rate of their efficiency" P. Massé [4, p. 48] solves the simplified problem of optimizing the size of capital investments. He takes on the problem of "single investment − multiple results", i.e. at the beginning of the period under study, capital of size I is "instantly" invested, then annually for an infinite period of time, the entrepreneur receives annually the same income $f(I)$, regardless of the period of operation. This assumption greatly simplifies the mathematical derivation and its graphical illustration. The assumption of an infinite service life is not a significant limitation. A more important assumption is that the curve $f(I)$ is upward convex, i.e. the law of decreasing investment efficiency is observed, as shown below in Fig. 1: net annual income takes negative values in the initial section of the $f(I)$ curve, i.e. when investments are below a certain threshold I_{min}, and the values of the function $f(I)$ begin to decrease when investments exceed a certain expedient amount I_{max}. The analysis concerns the section of the curve $f(I)$ between two investment volumes I_{min} and I_{max} (Fig. 2).

In this particular simplified case, the discounted total profit NPV is equal to the below proved equation

$$NPV = f(I)/E − 1 \tag{3}$$

and the internal rate of return (below proved)

$$IRR = f(I)/I. \tag{4}$$

These two formulas are mentioned by P. Masse without proof. He identifies two characteristic points M and N (Fig. 1). At point M, the maximum IRR indicator is reached, and at point N, the maximum NPV indicator is reached. Since the net annual income at point N is greater than at point M, Masse concludes from here that the NPV criterion is preferred over the IRR criterion. Are there sufficient

grounds for such an assertion? To comprehend Masse's statements, one should understand formulas (2) and (3).
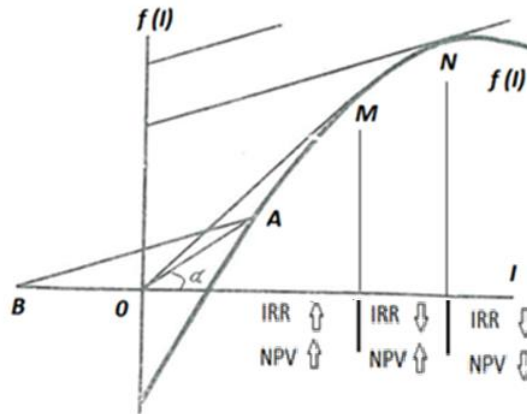

Fig. 1. **Analysis of NPV and IRR criteria (by Masse)**

The proof of (3). According to formula (1), NPV is in this particular case as the following

$$NPV = \sum_{t=1}^{\infty} f(I)\frac{1}{(1+E)^t} - I\frac{1}{(1+E)^0} = \frac{f(I)}{1+E}\sum_{t=0}^{\infty}\frac{1}{(1+E)^t} - I$$

Since for

$$\sum_{t=0}^{\infty} q^t = \frac{1}{1-q}$$

at $q < 1$, then, denoting $1/(1 + E) = q$, we have

$$\sum_{t=0}^{\infty}\frac{1}{(1+E)^t} = \frac{1+E}{E}$$

Therefore, $NPV = f(I)/E - 1$, and formula (3) is proved.

The proof of (4). Substituting the value of I and f(I) into formula (2), we have

$$\frac{-I}{(1+IRR)^0} + \sum_{t=1}^{\infty} f(I)\frac{1}{(1+IRR)^t} = 0$$

Since

$$\sum_{t=1}^{\infty}\frac{1}{(1+IRR)^t} = \frac{1}{IRR}$$

then $-I + f(I)/IRR = 0$, and formula (4) is proved.

Geometric interpretation of NPV and IRR formulas

Fig. 2 gives a geometric interpretation of formulas (3) and (4) in more detail. Let us take some amount of investment I_a and find the corresponding point A on the curve $f(I)$. Let us draw a segment OA from the origin 0, which forms an angle α with the abscissa axis, and another segment BA such that it forms the angle β and $tg\,\beta = E$ is true, where E appears in (3).

Let us prove that for the chosen value

$$I_a = |B0|, \tag{5}$$

where $|B0|$ – denotes the length of the segment $B0$.

Indeed, since $tg\,\beta = f(I_a)/|B0| + I_a$, then $|B0| = f(I_a)/E - I_a$. Thus, taking into the account (3), relation (5) is proved. Further, $tg\,\alpha = f(I_a)/I_a$. Consequently, from formula (4) it follows that $tg\,\alpha = IRR$.
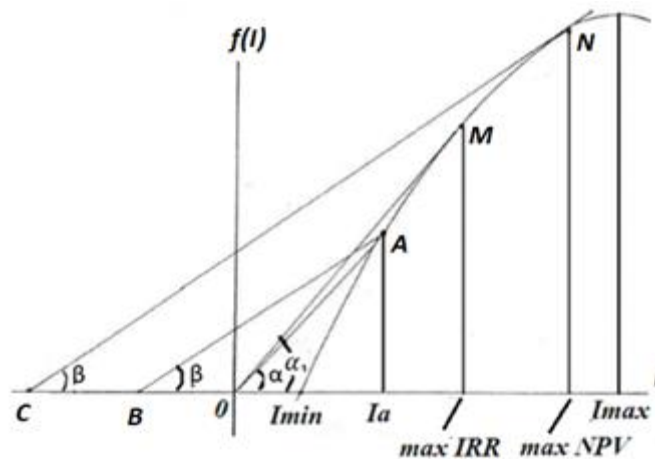
Fig. 2. **Detailed analysis of NPV and IRR criteria**

Following the learned geometric interpretation of the NPV and IRR criteria, we find that the maximum value of the NPV criterion corresponds to point N. At this point, the $f(I)$ curve has a tangent CN with the same angle β, and the length of the segment $C0$ (i.e. the value of the criterion NPV) takes on a maximum value. The maximum value of the IRR criterion occurs somewhat earlier − at point M, where the tangent to the curve $f(I)$ passes, drawn from the origin 0 with angle α_1.

Analysis of IRR and NPV

Thus, three intervals can be distinguished on the $f(I)$ curve (see Fig. 1):

1. to the left of M, where both NPV and IRR criteria increase with I;
2. between M and N, where NPV increases with I but IRR decreases;
3. to the right of N, where both NPV and IRR decrease with the increase of I.

Based on the analysis of the considered example, P. Masse draws two important conclusions:

Conclusion 1. The optimum at the point N (i.e. the NPV criterion) should be preferred, since the second optimum at the point M leads to Malthusian solutions in the problems of optimizing the size of capital investments.

Conclusion 2. Smaller investments with a higher IRR should be preferred to investments of a larger volume with a lower IRR, if they allow to earn a large discounted profit NPV.

Can we agree with these arguments? Should practical investment activity be guided by the NPV criterion, and not by the IRR criterion? Far from it.

In the first conclusion, it is proposed to follow the NPV criterion because of the danger of obtaining Malthusian solutions when following the IRR criterion. What kind of Malthusian solutions are meant, unfortunately, there are no clarifications or references in the text of the book. This, apparently, is about the well-known reasoning of T.R. Malthus (1766-1834) about the geometric growth of population − compared with the arithmetic growth of national wealth. Indeed, from the point of view of the interests of society, it would not be worthwhile to refuse investments for which the value of the IRR criterion, although less than the maximum value achieved at point M, still exceeds the bank interest rate E. It could be reasonable from the view of Macroeconomics, from the view of nation's interest. But what can make an entrepreneur follow such a rule − to invest in this enterprise? It is more profitable for him to confine himself to investments of the size $I_m = \max IRR$ (Fig. 2), and to invest the rest of the capital in excess of the size I_m, if possible, in an equally profitable business. If the state, with the support of a credit or tax system, is of interest to an entrepreneur, then by doing so it increases the value of the IRR indicator. Then the reference to Malthus has nothing to do with the investment activity of the entrepreneur.

As for the second conclusion of P. Masse, the answer is obvious: this statement is true only in one particular case − in conditions of infinite capital, which is not directly related to the optimization of investments by an entrepreneur, since the limited funds of an investor is an axiom of economics.

Thus, we conclude that the reasoning of P. Masse does not prove the advantage of the NPV criterion over the IRR criterion.

On the fallacy of Masse's IRR reasoning

Discussing the application of the IRR criterion, P. Masse writes [4, p. 54]: "The concept of relative efficiency IRR is used when an enterprise, having already a production program, must choose the optimal equipment". This concerns the relative effectiveness of the two projects. A numerical example shows the fallacy of Masse's reasoning.

Masse proposes to characterize the relative efficiency by the difference of two expressions of type (2) with the same unknown indicator IRR, namely: by the following equation

$$z^2 - z^1 = (P^2_1 - P^1_1)/(1 + IRR) + (P^2_2 - P^1_2)/(1 + IRR)^2 + \ldots \tag{6}$$

Here z^1 and z^2 are single investments for the first and second projects, respectively, but P^1_i and P^2_i are the results by years i.

A simple numerical example shows the meaninglessness of equation (6). Let there be two projects: $z^1 = 1$, $P^1_1 = 2$, $z^2 = 8$, $P^2_1 = 10$, others P^1_i and P^2_i are equal to zero. From equation (2) we have $z = P/(1 + IRR)$. This for the first and second project respectively gives $IRR_1 = 1$, $IRR_2 = 0.25$. Therefore, the internal rate of return IRR of the first project is four times higher than for the second one.

At the same time in coincidence with Masse's proposal, the indicator of relative efficiency according to formula (6) shows the opposite, namely, by substituting these values of costs and results into this formula, we have $8 – 1 = (10 – 2)/(1 + IRR)$, therefore $IRR = 0.14$.

How can one agree with this if, when considering the projects separately, the opposite conclusion is obtained? This numerical example shows the fallacy of Masse's reasoning and indicates unequivocal choice among the criteria NPV and IRR.

Discussion

Our goal here is about the statement "IRR performs better than NPV" but, honesty speaking, it is an unsolved question really and is relevant to this day. This inconsistency sparked a debate about which criterion is better. The debate has lasted for more than 100 years. Proof of this is the abundance of scientific papers: Google Scholar gives about 6 million references on request "IRR vs NPV" totally and even 16400 references on request "IRR vs NPV mathematical analysis" since 2019 only.

Classical works of prominent economists, containing the most complex mathematical reasoning about the effectiveness of investments (refer, for example, Fisher [6], 1906), put pressure on contemporaries till now. As a particular case of such an influence, let us name the paper of A. Jaunzems [7], the highly skilled in mathematical economics. Under Fisher's influence, he explained that the problem about priority of either NPV or IRR does not exist because of substantial difference between these concepts. The most important tool for cash flow analysis is function $NPV(i)$, $i \geq 0$, as a whole. In the same work, a few lines later, he refers to K. K. Seo [8], who wrote that approximately 55% of business persons evaluate the investment projects with help of IRR and only 10% − with help of NPV. It can be easily explained if we allow that the bank interest relates to Macroeconomics, largely concerned with nation's scale projects but the entrepreneur interest relates to Microeconomics, to the internal rate of return. The fallacy of the above discussed Masse's case also indicates a unequivocal choice among the criteria NPV and IRR.

In this sense, noteworthy is the work of Kannapiran Arjunan [9]. This paper presents contemporary data for determining an appropriate investment criterion (IRR vs NPV) focusing on the controversial reinvestment assumption, multiple, negative, zero or no IRR, mutually exclusive investments as well as independent projects. The analysis is based on estimated return on equity (ROC), return on invested capital (ROIC), depreciation schedule of capital (CAS) and modified CAS (MCAS). These results consistently support the conclusion that IRR is the best criterion for accepting or rejecting or ranking mutually exclusive projects as well as independent projects. NPV will continue to be useful in other areas for present value estimation. The proposed new method solved most of the problems such as reinvestment, better ranking of mutually exclusive projects by IRR and why the NPV rules need to be revised. At the same time, it shows that no unambiguous solution has been found yet.

Both NPV and IRR are sound analytical tools. However, they do not always agree and tell us what we want to know, especially when there are two competing projects with equally favourable alternatives.

Conclusions

1. The proof of both IRR and NPV formulas (in a particular simplified case) and geometric interpretation of these very complex equations are given (useful for the training purpose, at least).
2. The analysis of IRR and NPV indicates unequivocal choice among the criteria NPV and IRR. This confirms a simple numerical example on the fallacy of Masse's IRR reasoning.
3. No unambiguous solution has been found yet. It can be explained if we allow that the bank interest relates to Macroeconomics, largely concerned with nation's scale projects but the entrepreneur interest relates to Microeconomics, to the internal rate of return. The world continues to search for a single consistent criterion for evaluating investments.

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